Comparison Tests:

Example 1.

Use the Direct Comparison Test to determine whether the series

$\sum_{n=1}^{\infty} \frac{3}{n^3+7}$ converges or diverges.

Compare this series with $\sum_{n=1}^{\infty} \frac{3}{n^3}$ which converges. Since $\frac{3}{n^3+7} < \frac{3}{n^3}$ for all n, $\sum_{n=1}^{\infty} \frac{3}{n^3+7}$ converges.

Example 2.

Use the Direct Comparison Test to determine whether the series

$\sum_{n=1}^{\infty} \frac{3}{4+6^n}$ converges or diverges. Compare this series with $\sum_{n=1}^{\infty} \frac{3}{6^n}$ which is a geometric series with $r = \frac{1}{6}$. Since $\frac{3}{4+6^n} < \frac{3}{6^n}$ for all n, $\sum_{n=1}^{\infty} \frac{3}{4+6^n}$ converges.

Example 3.

Use the Direct Comparison Test to determine whether the series

$\sum_{n=1}^{\infty} \frac{7}{\sqrt{n}}$ converges or diverges. Compare with $\sum_{n=}^{\infty} \frac{7}{n}$

which is 7 times the harmonic series & diverges. Since $\frac{7}{\sqrt{n}} > \frac{7}{n}$ for all n,

$\sum_{n=1}^{\infty} \frac{7}{\sqrt{n}}$ diverges.

Example 4.

Use the Limit Comparison Test to determine whether the series

$\sum_{n=1}^{\infty} \frac{7n+5}{n(n-7)(n-5)}$ converges or diverges. This series is

much like $\sum_{n=1}^{\infty} \frac{7}{n^2}$ which converges.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{7n+5}{n(n-7)(n-5)} \cdot \frac{n^2}{7}$$

$$= \lim_{n\to\infty} \frac{7n+5}{n} \cdot \frac{n}{n-7} \cdot \frac{n}{n-5} \cdot \frac{1}{7} = 7 \cdot \frac{1}{7} = 1.$$

So $\sum_{n=1}^{\infty} \frac{7n+5}{n(n-7)(n-5)}$ converges.